

Class-BA/B.Sc II (Sem. IV)

Subject - Mathematics

Paper- I (Statics and Solid Geometry)

Time Allowed : 3 Hrs

Maximum Marks : 50

Note :- Attempt any five questions selecting atleast two from each section.

Section - A

1. (a) The greatest and least resultant that two forces can have are of magnitude P and Q respectively. Show that when they act at an angle θ , their resultant is of magnitude

$$\sqrt{P^2 \cos^2 \frac{\theta}{2} + Q^2 \sin^2 \frac{\theta}{2}}$$

- (b) Two forces P and Q acting parallel to the length and base of a smooth inclined plane respectively. Would each of them singly support a weight W on

the plane, prove that $\frac{1}{p^2} - \frac{1}{Q^2} = \frac{1}{W^2}$ (5.5)

2. (a) Forces P, 3P, 2P, 5P act along the sides AB, BC, CD and DA of square ABCD. Find the magnitude and direction of their resultant and prove that it meets AD produced at point E such that AE:ED = 5:4.

- (b) A ring of weight W which can slide freely on a smooth vertical circle, is suspended by a string

- attached to highest point. If the thread subtends an angle Q at the centre, find the tension in the thread and the reaction of circle on the ring. (5,5)
3. (a) State and prove Varignon's theorem.
 (b) ABC is triangle and G is its centroid. A force R acts along AG. Resolve R into two forces parallel to it and acting at B and C respectively. (6,4)
4. (a) A weight can be just supported on a rough inclined plane by a force P acting along the plane or by a force Q acting horizontally. Show that the weight is $\frac{PQ}{(Q^2 \sec^2 \lambda - P^2)^{1/2}}$, λ being the angle of friction.
- (b) A uniform rod rests in a vertical plane with in a rough hemispherical bowl whose radius is equal to the length of the rod. If μ is the coefficient of friction between the rod and the bowl; Show that in limiting equilibrium the inclination of the rod to the horizontal is $\tan^{-1} \left(\frac{4\mu}{3-\mu^2} \right)$. (5,5).

Section - B

5. (a) Find the equation of cylinder whose generators are parallel to line $\frac{x}{l} = \frac{y}{m} = \frac{z}{n}$ and intersect the conic $ax^2 + 2hxy + by^2 = 1, z = 0$.

- (b) Find the equation of quadric cylinder with generators parallel to x-axis and passing through the curve $ax^2 + by^2 + cz^2 = 1$, $lx + my + nz = P$. (5,5)
- 6.(a) Find the equation of the enveloping cylinder of sphere $x^2 + y^2 + z^2 + 2x + 2y + 2z + 2 = 0$ and whose generators are parallel to the line $\frac{x}{1} + \frac{y}{-1} + \frac{z}{1}$.
- (b) Prove that the plane $ax + by + cz = 0$ cuts the cone $yz + zx + xy = 0$ in perpendicular lines if $\frac{1}{a} + \frac{1}{b} + \frac{1}{c} = 0$. (5,5)
- 7.(a) The plane $\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$ meets the co-ordinate axes in A, B, C. Prove that the equation of cone generated by the line drawn from O to meet the circle ABC is $yz \left(\frac{b}{c} + \frac{c}{b} \right) + zx \left(\frac{c}{a} + \frac{a}{c} \right) + xy \left(\frac{a}{b} + \frac{b}{a} \right) = 0$.
- (b) Find the equation of the cone whose vertex is at the origin and guiding curve is $\frac{x^2}{4} + \frac{y^2}{9} + \frac{z^2}{1} = 1$, $x + y + z = 1$. (5,5)

8. (a) If $\frac{x}{1} = \frac{y}{z} = \frac{z}{1}$ represents one set of three mutually perpendicular generators of cone $11yz + 6zx - 14xy = 0$. Find the equation of other two.

(b) Prove that the equation

$$ax^2 + by^2 + cz^2 + 2ux + 2vy + 2wz + d = 0 \text{ represents}$$

$$\text{a cone iff } \frac{u^2}{a} + \frac{v^2}{b} + \frac{w^2}{c} = d. \quad (5,5)$$
